The groups of symmetric genus $\sigma \leq 8$

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Abstract

Let $G$ be a finite group. The symmetric genus $\sigma(G)$ is the minimum genus of any compact Riemann surface on which $G$ acts. We obtain the following general results about the partial presentations that groups acting on surfaces must have.

Theorem. Let $G$ be a group of order $2^nm$, where $m$ is odd and $\sigma(G) \geq 2$. Suppose that $S$ is a Sylow 2-subgroup of $G$. Let $D$ be a maximal order dihedral subgroup of $S$ with $D = 1$ if $S$ has no dihedral subgroups. Let $C$ be a maximal order cyclic subgroup of $S$. If $[S : D] \geq 4$ and $[S : C] \geq 4$, then $\sigma(G)$ is odd. Furthermore, if $G$ acts on a surface of even genus $g \geq 2$ and contains no elements of order $2^{n-1}$, then a Sylow 2-subgroup of $G$ must be isomorphic to one of three groups.

Then we classify the groups of symmetric genus $\sigma$, for all values of $\sigma$ such that $4 \leq \sigma \leq 8$. This is done by proving some “refined Hurwitz theorems,” such as the following.

Theorem. Let $G$ be a group with $\sigma(G) \geq 2$. If $\sigma(G) < 1 + |G|/8$, then $G$ has a partial presentation (with the relations fulfilled) of type $T$, $Q$, $FT$, $HT$, $FQ$ or $HQ$. Further, the Singerman subgroup condition is satisfied if $G$ has a partial presentation of type $FT$, $HT$, $FQ$ or $HQ$.

The software package MAGMA was employed to check if a specific group satisfied these partial presentations. The MAGMA library of small groups was used to identify the groups to be checked.

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