We start by plotting the planes.

```maple
with(plottools): with(plots): with(LinearAlgebra):
eqn1 := z=-6;
eqn2 := y=0;
eqn3 := y-x=4;
eqn4 := 2*x+y+z=4;
planes := implicitplot3d([eqn1, eqn2, eqn3, eqn4], x=-5..6, y=-1.7..1.5, 
z=-7..15, color=[blue, red, green, yellow], style=patchnogrid, 
transparency=.5, axes=normal):
display(planes);
eqn1 := z = 6
eqn2 := y = 0
eqn3 := y - x = 4
eqn4 := 2x + y + z = 4
```

Next we find the lines of intersection of the planes and plot them as well, so we see the pyramid outlined.
We also show the same plot with the faces as well:

```plaintext
> f1 := plot3d([(10-u)*t/2+(u-4)*(1-t), u, -6], t=0..1, u=0..6, transparency=0, color=blue, grid=[20,20], style=patchnogrid):
f2 := plot3d([u, 0, -6*t+(1-t)*(4-2*u)], t=0..1, u=-4..5, transparency=0, color=red, grid=[20,20], style=patchnogrid):
f3 := plot3d([u, u+4, -6*t+(1-t)*(-3*u)], t=0..1, u=-4..2,
From now on we plot just the parts of the planes that make the sides of the pyramid.
Rotate the plot to see all sides of the pyramid

> display([f1,f2,f3,f4,lines], axes=normal);
To set up the integral over this region, determine the slice at each height $z_0$. (Change the value of $z_0$ to change the location of the slice.)

```maple
> ft1 := plot3d([(10-u)*t/2+(u-4)*(1-t), u, -6], t=0..1, u=0..6,
  transparency=0.6, color=blue, grid=[20,20], style=patchnogrid):
ft2 := plot3d([u, 0, -6*t+(1-t)*(4-2*u)], t=0..1, u=-4..5,
  transparency=0.6, color=red, grid=[20,20], style=patchnogrid):
ft3 := plot3d([u, u+4, -6*t+(1-t)*(-3*u)], t=0..1, u=-4..2,
  transparency=0.6, color=green, grid=[20,20], style=patchnogrid):
ft4 := plot3d([t*(4-u)/2+(1-t)*(-u)/3, 4-u-2*(t*(4-u)/2+(1-t)*(-u)/3), u],
  t=0..1, u=-6..12, transparency=0.6, color=yellow, grid=[20,20],
  style=patchnogrid):
z0 := 1:
sl := plot3d([(4-z0-u*(12-z0)/18)*t/2+(u*(12-z0)/18-4)*(1-t), u*
  (12-z0)/18, z0],
  t=0..1, u=0..6, transparency=0, color=black, grid=[20,20],
  style=patchnogrid):
display([ft1, ft2, ft3, ft4, lines, sl], axes=normal);
display([lines, sl], axes=normal);
```
Now we want to set up the triple integral. We see that \( z \) runs from -6 to the \( z \)-coordinate at which eqn2, eqn3, and eqn4 meet.

\[
> \text{solve}\{\text{eqn2,eqn3,eqn4}\};
\]
\[
\{x = -4, y = 0, z = 12\}
\]

So \( z \) runs from -6 to 12.

At a given height \( z \), we note that if we integrate first in \( x \) and then in \( y \), \( y \) runs from 0 to the intersection of eqn3, eqn4, and the plane at height \( z \).

\[
> \text{solve}\{\text{eqn3,eqn4}\},\{x,y\};
\]
\[
\left\{x = \frac{-1}{3}z, y = 4 - \frac{1}{3}z\right\}
\]

So \( y \) runs from 0 to \( 4 - z/3 \).

Finally, for a given value of \( y \) and \( z \), \( x \) runs from the back (green) plane to the front (yellow) plane:

\[
> \text{solve}\{\text{eqn3}\},\{x\};
\]
\[
\text{solve}\{\text{eqn4}\},\{x\};
\]
\[
\{x = y - 4\}
\]
\[
\left\{x = \frac{1}{2}y - \frac{1}{2}z + 2\right\}
\]
Add the range of integration for $x$ to the plot for a fixed height $z_0$ and value $y_0$ of $y$ (change $y_0$ to anything from 0 to 4-$z_0/3$).

$$y_0 := 1.5;$$

```
xr := spacecurve([t*(y0-4)+(1-t)*(2-y0/2-z0/2), y0, z0], t=0..1, color=white, thickness=3);
display([ft1, ft2, ft3, ft4, lines, sl, xr], axes=normal);
display([lines, sl, xr], axes=normal);
y0 := 1.5
```
So here's the integral that computes the volume of the pyramid:

\[
> \int \int \int \frac{4}{3} - \frac{1}{2} y - \frac{1}{2} z + 2 \, dx \, dy \, dz
\]

\[
\int_{-6}^{12} \int_{0}^{4-z/3} \int_{-4}^{2-y/2-z/2} 1 \, dx \, dy \, dz
\]

Compute in steps:

\[
> \int \int \int (\frac{4}{3} - \frac{1}{2} y - \frac{1}{2} z + 2) \, dx \, dy \, dz - \int \int \int \left( -\frac{3}{2} y - \frac{1}{2} z + 6 \right) dy \, dz
\]

\[
\int_{-6}^{12} \int_{0}^{4-z/3} \int_{-4}^{2-y/2-z/2} \frac{4}{3} - \frac{1}{2} y - \frac{1}{2} z + 2 \, dx \, dy \, dz
\]

\[
\int_{-6}^{12} \int_{0}^{4-z/3} \int_{-4}^{2-y/2-z/2} \left( -\frac{3}{4} \left( 4 - \frac{1}{3} z \right)^2 - \frac{1}{2} z \left( 4 - \frac{1}{3} z \right) + 24 \right) dy \, dz
\]
\[ -2z \] \, dz \\
> \text{Int(Int(Int(1, x=y-4\,..\,2-y/2-z/2),y=0\,..\,4-z/3),z=-6\,..\,12)} = \text{int(int(int(1, x=y-4\,..\,2-y/2-z/2),y=0\,..\,4-z/3),z=-6\,..\,12));} \\
\int_{-6}^{12} \int_{0}^{12} \int_{y-4}^{4 - \frac{1}{3} z - \frac{1}{2} y - \frac{1}{2} z + 2} 1 \, dx \, dy \, dz = 162