

## A few useful Maple commands

Define zeta as a root of its minimal polynomial

$$\begin{aligned} > \text{Phi13} := \text{sum}(x^i, i=0..12); \\ \Phi_{13} &:= 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + x^{11} + x^{12} \end{aligned} \quad (1)$$

$$\begin{aligned} > \text{alias}(zeta = \text{RootOf}(\text{Phi13})); \\ & \zeta \end{aligned} \quad (2)$$

$$\begin{aligned} > \text{zeta}^{15}; \\ & \zeta^{15} \end{aligned} \quad (3)$$

The evala() (evaluate as an algebraic number) command may be needed.

$$\begin{aligned} > \text{evala}(\text{zeta}^{15}); \\ & \zeta^2 \end{aligned} \quad (4)$$

Check a calculation:

$$\begin{aligned} > \text{sum}(\text{zeta}^i, i=1..12); \\ & \zeta + \zeta^2 + \zeta^3 + \zeta^4 + \zeta^5 + \zeta^6 + \zeta^7 + \zeta^8 + \zeta^9 + \zeta^{10} + \zeta^{11} + \zeta^{12} \end{aligned} \quad (5)$$

$$\begin{aligned} > \text{evala}(\text{sum}(\text{zeta}^i, i=1..12)); \\ & -1 \end{aligned} \quad (6)$$

Compute a period. In this field extension the automorphism group has as one generator the map sending  $\zeta$  to  $\zeta^2$ . For the period whose extension has degree 2 over the rationals we want the sum of the even powers of this generator.

$$\begin{aligned} > \text{p2} := \text{evala}(\text{sum}(\text{zeta}^{(2^{(2*i)})}, i=0..5)); \\ p_2 &:= -1 - \zeta^2 - \zeta^5 - \zeta^6 - \zeta^7 - \zeta^8 - \zeta^{11} \end{aligned} \quad (7)$$

Sometimes Maple displays results in a counterintuitive form. If you see an expression with all coefficients negative we would write the result as the sum of the missing powers of  $\zeta$ .

Maple can find minimal polynomials using the PolynomialTools package.

$$\begin{aligned} > \text{with}(\text{PolynomialTools}); \\ > \text{MinimalPolynomial}(p_2, 2); \\ & -3 + \_X + \_X^2 \end{aligned} \quad (8)$$

For other problems it may be useful to factor polynomials over finite fields.

$$\begin{aligned} > \text{restart}; \\ > \text{f}(x) := x^7 - 14x^5 + 56x^3 - 56x + 22; \\ & f(x) := x^7 - 14x^5 + 56x^3 - 56x + 22 \end{aligned} \quad (9)$$

$$\begin{aligned} > \text{Factor}(f(x)) \text{ mod } 3; \\ & (x^3 + 2x + 1)(x + 2)(x^3 + x^2 + 2) \end{aligned} \quad (10)$$

$$\begin{aligned} > \text{f}(x) := x^6 + 24x - 20; \\ & f(x) := x^6 + 24x - 20 \end{aligned} \quad (11)$$

$$\begin{aligned} > \text{Factor}(f(x)) \text{ mod } 23; \\ & (x + 21)(x + 12)(x^3 + 6x^2 + 13x + 16)(x + 7) \end{aligned} \quad (12)$$